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# OBSERVATIONS

ON

# THE HYPOTHESES

WHICH HAVE BEEN ASSUMED TO ACCOUNT FOR

THE

# CAUSE OF GRAVITATION

FROM

MECHANICAL PRINCIPLES.

BY

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CAMBRIDGE:
*Printed by R. Watts, at the University Press;*
AND SOLD BY DEIGHTON, AND NICHOLSON, CAMBRIDGE;
AND BY LUNN, SOHO-SQUARE, LONDON.
1806.

*Price One Shilling.*

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# PREFACE.

SIR I. NEWTON thought the refutation of DES CARTES's doctrine of vortices, deserving a place in his *Principia*; but his arguments there advanced are esteemed by no means satisfactory. What is here stated in Art. 6., appears to be conclusive against the truth of any system which is founded upon such a principle; and might indeed have superseded the necessity of any further enquiry upon the subject, had it been thought of before the other part was drawn up. In his *Optics*, Sir I. NEWTON attempts to account for gravity by means of an elastic fluid; this, however, he proposes by way of a question, not being satisfied about it, as he says, for want of experiments; these, however, he never made; nor has any one since examined his hypothesis, in order to discover whether it will account for the *law* of gravitation; for it is not sufficient merely to show that such a medium may exist as will drive a body *towards* the sun*. What Sir I. NEWTON left for further examination, will be deemed no impertinent nor useless enquiry; more particularly at this time, when many of the most eminent Philosophers upon the Continent have been endeavouring to account for all the operations of nature upon merely mechanical principles, with a view to exclude the Deity from any concern in the government of the system, and thereby to lay a foundation for the introduction of Atheism. Upon this account, the author was requested to consider the subject, and give the result of his examination. The enquiry was favourably received; and it was suggested, that it might not be improper to be offered to the Royal Society. It was accordingly submitted to the consideration of the ASTRONOMER ROYAL, and he presented it to the Society; when the

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* Mr. MACLAURIN observes, that this hypothesis no way derogates from the government and influences of the Deity, whilst it leaves us at liberty *to pursue our enquiries concerning the nature and operations of such a medium.* And Sir J. PRINGLE, the late *worthy* and *learned* President of the Royal Society, who executed the duties of his high office with great *impartiality* and *honour,* considering the importance of the subject, recommended it as deserving the attention of philosophers.

President and one of the Secretaries requested that the author would withdraw it, and present it again in the November following, as the paper appeared a proper subject for the *Bakerean* Lecture. It was accordingly withdrawn, and offered again at the time when it was requested to be presented. The paper was then read, and appointed to be the *Bakerean* Lecture. But *before* it went into the Committee which is expressly appointed to examine and determine what papers shall be printed, the author was informed that it was doubtful whether his paper would be published. The circumstances attending this information, led him to suspect that it would not appear in the Transactions of the Society, and in this he was not disappointed. As the subject however is curious in itself, and appears in it's consequences to be of great importance, the author thinks it may not be improper to submit the result of his enquiry to the Public.

# OBSERVATIONS,

## *&c. &c.*

1. Two bodies at a distance can act upon each other, only by the intervention of some third body; for a body cannot immediately act where it is not. Thus, two distant bodies *A* and *B* may act upon each other by an intermediate body connecting them, as two bodies act upon each other by the arms of a lever to which they are attached; or, they may act upon each other by means of an intermediate body *C*, as *A* may act upon *C*, and *C*, by the motion communicated to it, may act upon *B*. Also, we perceive the sun by the propagating of it's rays to us; we perceive the sound of a bell by the intervention of air. These, and such like, we call the mechanical actions of bodies upon each other.

2. Two bodies are said to *attract* each other, when they approach each other without any visible cause: thus, when a stone is let go from the hand, it moves towards the earth without any apparent cause, and the earth is said to attract the stone. Two bodies

are said to *repel* each other, when they recede from each other without any visible cause: thus, if two bodies be electrified with the same power, they recede from each other without any visible cause, and they are said to repel each other. If these operations therefore take place from mechanical causes, we must seek for some intermediate body; and the nature of this intermediate third substance, if such there be, is the subject of our present enquiry.

3. Let us take the case of the sun attracting the planets; and these being distant bodies, some intermediate third body must be sought for, in order to account for the effect produced, supposing it to be produced by mechanical causes.

4. KEPLER supposes that the effect of attraction is produced by an immaterial image of the sun, projected from the sun to the planet, and drawing the planet towards the sun. He says, "Effluxus igitur, quemadmodum et lucis, immateriatus est; non qualis odorum cum diminutione substantiæ, non qualis caloris ab æstuante fornace, et si quid est simile, quibus media implentur. Relinquitur igitur, ut quemadmodum lux, omnia terrena illustrans, species est immateriata ignis illius, qui est in corpore solis: ita virtus hæc, planetarum corpora complexa et vehens, sit species immateriata ejus virtutis, quæ in ipso sole residet, inæstimabilis vigoris, adeoque actus primus omnis motus mundani.—Videri namque possit in corpore solis latitare divinum quippiam, et comparandum animæ nostræ, ex quo effluat species ista planetas circumagens, uti ex anima jaculantis lapillos species motus in lapillis adhærescit, qua provehuntur illi, etiam cum qui jaculatus est manum ab illis reduxit." *(De Motibus Stellæ Martis.)*—"Etsi igitur species corporis solaris attenuatur in longum et latum,

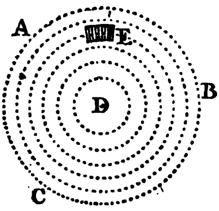
non minus quam lux—videtur potius hoc sequendum, effluere speciem immateriatam corporis ipsius, cui speciei et vis prensandi, et lux, luci vero et calor et color, quodlibet ex suo fonte derivatum, inhæreant—nec enim uspiam est species illa, nisi in opposito et occurrente corpore, lucis quidem in ejus superficiei opaca; virtutis vero motricæ in tota corpulentia: in spatio vero intermedio inter solem et superficiem, non est, sed fuit." *(Epist. Astron.)* It is not here easy to annex any clear ideas to our author's meaning; in what manner this immaterial image of the sun acts upon the planet to bring it towards the sun (vis prensandi) he has not explained. He speaks of the cause as an immaterial active body; as a divine mind; as the mind of man; and by it's action retaining the planets in their orbits. The unintelligibility of this precludes all examination; and nothing but it's being the hypothesis of so great a man, could justify our producing it. But whatever may have been his meaning, it can have nothing to do with our present enquiry, which is to consider how far gravity can be accounted for from the operation of mechanical causes.

5. The only hypotheses which have met with any advocates amongst philosophers, are those of Des Cartes, Sir I. Newton, and I. Bernoulli. Des Cartes attempted to account for the motions of the planets, by supposing them to be carried round the sun in vortices. But Sir I. Newton, besides his other arguments against this theory, very justly observes, that the free motions of comets in all directions, and their being subject to the same laws which the planets are, destroy at once the hypothesis of vortices. Sir I. Newton accounts for gravitation in the following manner. He supposes that there

is a fluid surrounding the sun, which increases in density as it recedes from the sun, and that a body put into this fluid will be forced towards the sun by the difference of the pressures on the opposite sides. I. Bernoulli was dissatisfied with both these hypotheses, and formed a system from the two (un juste milieu entre les deux m'a paru le plus sur), which he thinks will solve all the phenomena. He supposes that there are two kinds of vortices, one belonging to the sun, and the other to the principal planets. He conceives also, that there is a torrent which sets in continually from the extremity of the vortex towards the center, causing thereby a tendency of all the bodies in the vortex towards that point, and thus he accounts for the gravitation of the planets to the sun. He further supposes that there are two sorts of matter; one, perfectly fluid without any elementary particles; without inertia; and which gives no resistance to bodies moving in it; the other formed of corpuscles. These two matters are mixed together, and form the vortex. When the vortex first began to revolve, the particles of the second kind of matter receded a little from the center and formed a vortex of different densities, and thus there remained a space about the center of the vortex, filled with the first matter only, and this is the matter which forms the sun. The matter of the sun being in an effervescent state, small particles are thrown off (as steam from boiling water), and as these must act upon each other and upon the particles of the second kind of matter, many of them must constantly be reflected back, and thus a current (as before mentioned) is continually setting in from the extremity of the vortex towards the center. The particles thus thrown off, form both light and heat. Such is the system according to the hypothesis of this author. But he sets out with a supposition which can never be admitted, and upon the truth of which his investigations depend: that is, that the different parts of the fluid, as it revolves about the sun, act upon each other in proportion to their distances from the sun, in the same manner as if they were connected by an inflexible lever revolving about the same center. It is unnecessary to enter into any examination of this principle; to show it's absurdity, we may observe, that, upon this ground, if one particle of the fluid were stopped, the whole should cease to revolve, such being the property of bodies connected by a lever. But this is contrary to the nature of all fluids with which we are acquainted, and repugnant to every idea which we have of any fluid, either elastic or non-elastic. And the effect here supposed to take place, is certainly less likely to happen in a mixture of two fluids, one of which is so perfect as to give no resistance, than in the fluids with which we are acquainted, and which we know, from experience, are not endued with any such property. Setting aside all other objections to which this theory is liable, his fundamental proposition can never be admitted. He objects to NEWTON's theory, that it will not account for the motion of the apsides; but it is known that it will account most accurately, not only for this motion, but also for all the irregularities which take place in the system. NEWTON's theory gives the true *quantity* of all these motions, but BERNOULLI does not attempt to investigate the *quantity* of the motion of the apsides, he only endeavours to show that a motion of the apsides will be produced upon his hypothesis. This might be admitted to be true; but unless the theory gave the *true quantity* of that motion, his hypothesis must still be false. **Newton's** law of gravitation was objected to at first, because it was supposed that it would not give the true quantity of the motion of the lunar apsides; but when it was found that it did give the true quantity, the law was admitted. **Bernoulli** also shows, that his theory makes the earth an oblong spheroid, as determined (he says) by the accurate measurement of **Cassini**. "D'ou je dois inferer, *suivant ma théorie,* que la figure de la terre est à la vérité, celle d'un spheroïde *allongé,* conformément au resultat des observations faites en France par des mesures actuelles.—Apres cette heureuse conformité *de notre théorie* avec les observations célestes, peut-on plus long-tems refuser à la terre la figure de spheroïde *oblong,* fondé d'ailleurs sur la dimension des degrés de la méridienne, entreprise et executée par le meme M. **Cassini**, avec *une exactitude inconcevable?*" His theory leading him to this false conclusion, is, of itself, a sufficient argument against it's admission.

6. **Huygens** also attempted to account for gravitation by means of vortices. He says, "fingamus circa centrum $D$ volvi materiam fluidam contentam spatio $ABC$, quo exire nequeat, propter alia circumjacentia corpora. Constat partes omnes fluidi illius conari recedere a centro $D$, sed frustra, quia nimirum partes, quæ deberent in illum locum succedere, non minus conantur a centro illo fugere. Verum si qua esset pars illius materiæ, puta $E$, quæ motum circularem cæterarum non sequeretur, aut quæ lentius moveretur quam aliæ, dico quod ea centrum versus impelletur. Quandoquidem enim aut non conatur aut minus conatur quam partes propinquæ ab eo recedere, cedet actioni earum, quæ minus distant a centro $D$, et ista locum dabit cæteris accedendo ad

illud, quoniam aliter locum ipsis cedere nequit." Upon this principle, the attraction towards the central body does not at all depend upon the quantity



of matter in that body, inasmuch as a body placed in the vortex may approach the center without any central body; a circumstance which is totally inconsistent with the principles of gravitation. And we may also further observe, that this argument is conclusive against the doctrine of vortices as delivered by DES CARTES; and it must equally prevail against the truth of any system founded upon such a principle.

7. M. SAUSSURE accounts for attraction, by supposing that all space is filled with particles of matter, moving rapidly in all directions; and that the particles which fall on the opposite sides of any two bodies, in lines parallel to the line joining their centers, will impel the bodies towards each other, the sides next each other not being so acted upon. But in this case, the moving force of each body would depend on the surfaces of the bodies; whereas it ought to be as the quantity of matter in each respectively. This consideration therefore, without entering into any further examination of this hypothesis, is sufficient to show that it cannot be admitted.

8. The gravitation of a planet towards the sun

varies inversely as the square of it's distance from the sun; that is, whatever be the magnitude or density of the planet, it's acceleration towards the sun varies in that ratio. Whatever cause therefore is assigned, it must satisfy this law of variation.

9. If we suppose material particles to emanate from the sun and act upon the surface of the planet, or to pervade the body and act upon the whole of it, the tendency of such action, from the known mechanical operations of bodies upon each other, must be to drive the planet *from* the sun. Such an emanation therefore cannot be admitted as the cause of gravitation. We will therefore next consider Sir I. NEWTON's hypothesis.

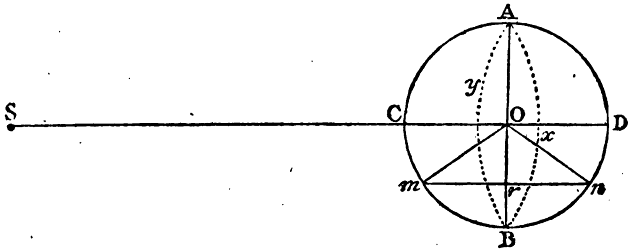
10. If the sun and planets act upon each other, it must be by some intermediate substance which is invisible; this substance we must therefore suppose to be an elastic fluid; and upon such a cause, Sir I. NEWTON attempts to account for gravitation. In his second advertisement to the second edition of his Optics, he says, "To show that I do not take gravity for an essential property of bodies, I have added one question concerning it's cause; chusing to propose it by way of a question, because I am not yet satisfied with it for want of experiments." He therefore proposes (Optics, Quæry 21.) to account for the gravitation of the planets towards the sun, "By means of an elastic fluid surrounding the sun (and in like manner surrounding every other body), supposing this medium to increase in density as it passes at greater distances from the sun, causing thereby the gravity of these greater bodies towards each other, and of their parts towards the bodies; every body endeavouring to go from the denser parts of the medium towards the rarer. For if this

medium be rarer within the sun's body than at it's surface, and rarer there than at the hundredth part of an inch from it's body, and rarer there than at the orbit of Saturn; I see no reason why the increase of density should stop any where, and not rather be continued at all distances from the sun to Saturn, and beyond. And though this increase of density may, at great distances, be exceeding slow, yet if the elastic force of this medium be exceeding great, it may suffice to impel bodies from the denser parts of the medium towards the rarer, with all that power which we call gravity." And he gives the following reason for supposing that there is such a medium. "A thermometer in vacuo will grow warm almost as soon as a thermometer which is not in vacuo. Is not the heat of a warm room therefore conveyed through the vacuum by the vibrations of a much subtiler medium than air, which, after the air was drawn out, remained in the vacuum?" (Quæry 18). Again, "Doth not the refraction of light proceed from the different densities of this ætherial medium, in different places, the light receding always from the denser parts of the medium towards the rarer? And is not the density thereof greater in free and open spaces void of air, and other gross bodies, than within the pores of water, glass, crystal, gems, and other compact bodies. For when light passes through glass or crystal, and falling very obliquely upon the further surface thereof is totally reflected, the total reflection ought rather to proceed from the density and vigour of the medium without and beyond the glass, than from the rarity and weakness thereof. And doth not this ætherial medium in passing out of water, glass, crystal, and other compact and dense bodies into empty spaces, grow denser and denser by degrees; and by that means refract the rays of light, not in a point, but by bending them gradually in curve lines? And doth not the gradual condensation of this medium extend to some distance from the bodies?" (Quæry 19, 20.) Making this fluid therefore the cause of the reflection, refraction and inflection of light, it must (as our author thinks) be denser without the bodies than within; and that this increase of density may be continued, not only so far as may be necessary to turn light out of it's course, but also through the heavens, and cause the gravitation of the bodies. We are therefore to enquire, whether the *law* of gravitation can be deduced from this cause.

11. A force which varies in the inverse duplicate ratio of the distance, cannot be compounded of several forces, each of which does not vary in that ratio. For let $a$ be the distance, and the forces to be represented by $\frac{s}{a^m}$, $\frac{t}{a^n}$, $\frac{v}{a^p}$, &c. and $\frac{d}{a^2}$, and suppose $\frac{s}{a^m}+\frac{t}{a^n}+\frac{v}{a^p}+\text{\&c.}=\frac{d}{a^2}$, then $sa^{2-m}+ta^{2-n}+va^{2-p}+\text{\&c.}=d$; now this equation cannot in general be true, that is, for every assumed value of $a$; for, when cleared of surds, it cannot have more roots than it has dimensions.

12. Let $S$ be the place of the sun, $ACBD$ a planet, $O$ it's center, and conceive $AxBy$ to be a plane perpendicular to $SCOD$, and let $mrn$ be perpendicular to the diameter $AOB$, and therefore parallel to $SCOD$. Put $a = SO$, $x = mr = nr$, $y = Or$, $z = Bm = Bn$, $r = Om$; then $a + x = Sn$, $a - x = Sm$, extremely nearly, so as not to produce any error which can possibly affect our conclusions;

and let $p = 6,283$ &c. Let the density of the fluid be measured by the number of particles uniformly



diffused on a given square area; and at any distance $d$ from $S$, let the density be as $d^m$; then the number of particles on the given area is as $d^m$, and therefore the number of particles in the side of that area is as $d^{\frac{m}{2}}$; hence, the distance of the particles is as $\frac{1}{d^{\frac{m}{2}}}$. Now let the elastic force of the particles vary as the $n^{\text{th}}$ power of their distance, or as $\frac{1}{d^{\frac{mn}{2}}}$; then the whole elastic force of all the particles in the given area (being as the number of particles and the force of each conjointly) will be as $\frac{d^m}{d^{\frac{mn}{2}}} = d^{\frac{2m-mn}{2}}$. Hence, at the distance $a + x$ the elastic force on a given area is as $\overline{a+x}\vert^{\frac{2m-mn}{2}}$, and at the distance $a - x$ it is as $\overline{a-x}\vert^{\frac{2m-mn}{2}}$; and on any other area, the elastic force will be as that area, and $\overline{a+x}\vert^{\frac{2m-mn}{2}}$ or $\overline{a-x}\vert^{\frac{2m-mn}{2}}$. conjointly. Now if this elastic fluid pervade the body, each particle *within* the body, by exerting it's force equally in all directions, can produce no effect upon the body; we have therefore only to consider the effect which the fluid will have upon the surface of the body, perpendicular to which the elastic force of the fluid must act. Now at the distance $x$ from the plane $AxBy$, the fluxion of the surface of each hemisphere $ACB$, $ADB$, is $py\dot{z} = pr\dot{x}$; hence, the fluxion of the elastic force of the fluid on $ADB$, in a direction perpendicular to the surface, is as $pr\dot{x} \times \overline{a+x}|^{\frac{2m-mn}{2}}$, or as $r\dot{x} \times \overline{a+x}|^{\frac{2m-mn}{2}}$

$$= r\dot{x} \times a^{\frac{2m-mn}{2}} + \frac{2m-mn}{2} \times rx\dot{x} \times a^{\frac{2m-mn}{2}-1},$$

omitting, for the present, all the other terms of the series, as being so extremely small in comparison with the two first; and by the resolution of forces,

$$r : x :: r\dot{x} \times a^{\frac{2m-mn}{2}} + \frac{2m-mn}{2} \times rx\dot{x} \times a^{\frac{2m-mn}{2}-1} :$$

$x\dot{x} \times a^{\frac{2m-mn}{2}} + \frac{2m-mn}{2} \times x^2\dot{x} \times a^{\frac{2m-mn}{2}-1}$ the fluxion of the force of the fluid on $ADB$ in the direction $nr$. In like manner, the fluxion of the force of the fluid on $ACB$, in the direction $mr$, is $x\dot{x} \times a^{\frac{2m-mn}{2}} - \frac{2m-mn}{2} \times x^2\dot{x} \times a^{\frac{2m-mn}{2}-1}$. Subtract this latter force from the former, and we get $\overline{2m-mn} \times x^2\dot{x} \times a^{\frac{2m-mn}{2}-1}$ for the fluxion of the whole force with which the planet is impelled towards the sun, the fluent of which is $\frac{2m-mn}{3} \times x^3 \times a^{\frac{2m-mn}{2}-1}$, which, when $x = r$, becomes $\frac{2m-mn}{3} \times r^3 \times a^{\frac{2m-mn}{2}-1}$ the whole force with which the planet is urged towards the sun.

13. But the force here determined is the *moving* force; now the *accelerating* force is as the moving force directly and the quantity of matter inversely; if therefore we put $e$ for the density of the planet, the quantity of matter is as $er^3$; and the accelerating force becomes $\frac{2m-mn}{3e} \times a^{\frac{2m-mn}{2}-1}$. Now this accelerating force is partly in terms of $e$ the density of the planet, and therefore it does not agree with the law of gravitation (Art. 8.), that law being independent of the density. But further, that this force may vary in the inverse duplicate ratio of the distance $a$ of the planet from the sun, we must assume $\frac{2m-mn}{2}-1=-2$, in which case $\frac{2m-mn}{3}=-\frac{2}{3}$, and this being the coefficient of the quantity which expresses the force of the body towards the sun, it indicates (by it's being negative) that the planet must always be driven *from* the sun by the pressure of this elastic fluid. Now the terms omitted in the series are comparatively so extremely small, that if they were considered they could make no sensible alteration in the result. But if they were considered, as each term would give a force varying according to a different power of the distance from that which is retained, they could not, (Art. 11.) taken altogether, give a force which varies in the inverse duplicate ratio of the distance. It is not possible therefore for any law of variation of the density of the fluid in terms of any single power of the distance from the sun, combined with any

law of variation of the elastic force of the particles varying according to any single power of their distance, to constitute a fluid which shall impel a planet towards the sun by a force which varies in the inverse duplicate ratio of it's distance from the sun.

14. But before we extend our enquiries to the more general laws of variation of the density of the fluid, and repulsive forces of the particles, let us attend to the consequences which result from our present supposition, the force being supposed to vary in the inverse duplicate ratio of the distance from the sun. The fluid being supposed to increase in density from the sun, $m$ must be positive. Now $n=\frac{2m+2}{m}$; $n$ must therefore be a positive quantity between 2 and *infinity*. The repulsive force of the particles therefore varies in some direct ratio of their distance; and as the body is driven *from* the sun, it is impelled from the rarer towards the denser parts of the medium; the rarer parts of the fluid have therefore a greater repulsive force than those which are more dense; the reason of which must be this; that in the rarer parts, each particle repelling with a greater force, a repulsion is produced which more than counterbalances that which arises from an increase of density. It is not true therefore, that a body put into an elastic fluid must necessarily be driven towards the rarer parts.

15. If we suppose the density of the fluid to decrease from the sun, then $m$ must be negative. Now $m=\frac{2}{n-2}$, the limit for $m$ being negative is therefore when $n$ is any negative quantity, or any positive quantity less than 2. If therefore the

particles repel each other with forces varying in any inverse ratio of their distances, or in any direct ratio less than the square, a fluid decreasing in density may give a law of force propelling a body from the sun, which shall vary inversely as the square of the distance, and driving the body from the denser towards the rarer parts of the medium.

16. If $2m - mn = 0$, the body will remain at rest; this therefore happens when $m = 0$, or when $n = 2$. If therefore the density of the fluid be uniform with any law whatever of repulsive force of the particles, or if the repulsive force of the particles vary as the squares of their distances with any law of variation of density, in each ease the body will remain at rest. A body may therefore remain at rest in an elastic medium of an unequal density.

17. That the body may be impelled *towards* the sun, we must make the coefficient $\frac{2m - mn}{3} =$ some positive quantity, $+ Q$; in which case the index $\frac{2m - mn}{2} - 1$ (denoting the law of force) $= \frac{3Q}{2} - 1$; the limit of the greatest negative index is therefore $- 1$. Hence, the limit of the law of force by which a body may be impelled *towards* the sun, is that of the inverse distance, or any direct ratio. And if the fluid increase in density, or $m$ be positive, $n$ must be less than $2$; but if the fluid decrease in density, or $m$ be negative, $n$ must be greater than $2$.

18. But to proceed. It may be supposed, that if the above assumed law of density of the fluid will not answer the required conditions, yet some other law of density which is compounded of different powers of the distance, may be made to agree with the law of gravity. Let us therefore represent the

density of the medium by $Pa^m + Qa^q + Ra^s +$ &c. for by taking a proper number of these terms, this quantity may represent any variation of density, depending upon the variation of the distance. Hence, according to the foregoing reasoning (taking only the two first terms of the series), the law of force tending to the sun is $P \times \frac{2m-mn}{3e} \times a^{\frac{2m-mn}{2}-1} + Q \times \frac{2q-qn}{3e} \times a^{\frac{2q-qn}{2}-1} + R \times \frac{2s-sn}{3e} \times a^{\frac{2s-sn}{2}-1} +$ &c. Now these being different powers of the distance $a$, the whole can never constitute a power which varies as $\frac{1}{a^2}$ (Art. 11.); and much less if we take in the terms which were omitted.

19. If we suppose the repulsive force of the particles to vary as different powers of their distance, that is, as $\frac{1}{d^{\frac{mn}{2}}} + \frac{1}{d^{\frac{mv}{2}}} + \frac{1}{d^{\frac{mw}{2}}} +$ &c. it appears by the same reasoning, that we can never obtain an accelerating force varying $\frac{1}{a^2}$, which shall impel a body towards the sun; and moreover, the density of the planet constitutes part of the law.

20. Hence it appears, that it is not possible for any law of variation of the density of the fluid in terms of the distance from the sun, combined with any law of variation of the repulsive force of the particles of the fluid in terms of their distance, which can satisfy the law of gravitation. And if we were to suppose the law of density to vary in terms of any other quantities besides the distance from the sun, such quantities must enter into the law of force, and

thereby make a still further deviation from the law of the inverse square of the distance. Considering the matter therefore only in a mathematical point of view, we are justified in rejecting this hypothesis as the cause of gravitation. But it may be proper further to consider, whether such a fluid could exist under all the circumstances which were supposed to be necessary for solving the phænomena.

21. The fluid surrounding the sun, as the supposed cause of gravitation, has a tendency towards the sun (a supposition hard to be admitted when itself is supposed to be the cause of gravity), or it has not. If it were to tend towards the sun, the denser parts would be nearer the sun, like the atmosphere of the earth. If the fluid have no tendency towards the sun (the only supposition which we can make under the supposed circumstances of the fluid), it's elasticity must extend it's bulk without limit, and the whole would, immediately at it's first formation, have been dissipated. It is not possible for an elastic fluid, void of gravity, and having room for expansion, to be confined within any finite bounds. If it be said that it may be extended through infinite space, it will not remove the difficulty. For the most that could be admitted under such an hypothesis, is, that the density of the fluid would be uniform. It does not appear upon what principle we can admit the existence of such a fluid as Sir I. NEWTON here supposes.

22. If we suppose attraction to be performed by an elastic medium surrounding each body, how can these media run one into the other, and not mix and form one medium, but continue to act as two distinct media? Experience teaches, that if we mix any two elastic media of the same kind, they imme-

diately coalesce and become one; and this must always be the effect, in consequence of the particles having all the same relation to each other. Let us suppose, for instance, the Earth and Jupiter to be each surrounded with it's medium. Now the effects of these media, as the supposed cause of attraction, are exactly the same, both in respect to quality and law of variation, and therefore we must suppose these media to be of the same nature; indeed, NEWTON does not suppose them to be different. We cannot therefore suppose these media to be dispersed through the same space, and yet be kept perfectly separate, and act each independently of the other. On the contrary, they must necessarily form one medium, and therefore not be competent to produce the phænomena.

23. But if we consider the cause of attraction, as assigned by Sir I. NEWTON, we must proceed further. An elastic fluid is supposed to surround the sun, and the particles of this fluid are supposed to repel each other. Some body therefore (Art. 1.) must be interposed between the particles of this fluid, in order that *they* may act upon each other. Following Sir I. NEWTON's idea, we must suppose each of these particles to be surrounded by a fluid, which, being repulsive and interposing between the particles of the first fluid, causes them to be kept at a distance. And we must here proceed again in the same manner, and suppose each particle of this last medium to be surrounded by a like medium; and thus we must go on till the particles come into contact. I can see no other consequence than this, upon supposition that attraction arises from the operation of mechanical causes, in the manner here supposed. But Sir I. NEWTON supposes the elastic medium to be 700,000

times rarer than air; in which case, the distance of the particles must be nearly 90 times greater than those of air. In a medium of this rarity, Sir I. NEWTON supposes that the planets may move without any sensible resistance. He therefore considers the particles of this fluid as being kept at their distance, without the introduction of any thing further material, for that would increase the density. He leaves therefore the same difficulty unsolved in respect to the action between two particles of this fluid, as between any two of the heavenly bodies. And we may also further observe, that as the resistance varies at least in as high a ratio as the square of the velocity, if we consider the rapid motions of the planets, it will appear, even upon this assumed rarity of the medium, that the resistance would very soon disturb the system.

24. Sir I. NEWTON, in the last quæry of his Optics, speaks of the system as controlled by mechanical causes; for he says, "all material things seem to be made of hard and solid particles, variously associated in the first creation by the council of an intelligent Agent. For it became him who created them, to set them in order. And if he did so, it is unphilosophical to seek for any other origin of the world, or to pretend that it might arise out of chaos, by the mere laws of nature; though being once formed, it may continue for many ages. For while comets move in very eccentric orbits in all manner of positions, blind fate could never make all the planets move one and the same way in orbits concentric, some inconsiderable irregularities excepted, which may have arisen from the mutual actions of comets and planets upon each other, and which will be apt to increase, till the system wants a reformation."

Hence it appears, that Sir I. Newton supposed the system might be *governed,* though not *formed,* by merely mechanical causes, and which, at times, would want the hand of the Creator to correct it, and set it again in order. From the causes therefore which he has assigned, he concluded that the system would occasionally want a reformation; but by the discoveries of modern Astronomers, it appears that the law of gravitation will correct the irregularities which the mutual attraction of all the bodies must necessarily produce. Such a law is the most perfect that can be conceived, and affords the strongest instance of design and wisdom in it's choice.

25. The power and wisdom of the Deity are no where so conspicuous as in his government of the heavenly bodies. We see nothing in the heavens which argues imperfection; but imperfection is always found in the operation of mechanical causes. Now it seems reasonable to admit a Divine agency, at that point, when all other means appear inadequate to produce the effect. And as mechanical operations, in whatever point of view they have been considered, do not appear sufficient to account for the *preservation* of the system (to say nothing of it's *formation*), we ought to conclude, that the Deity, in his government, does not act by such instruments, but that the whole is conducted by his more immediate agency, without the intervention of material causes.